2013/8/19 Math 136-40C Design Technology

Dina Spain

(P3)

Section R1: [Real Numbers]

Union, Intersection + Complement

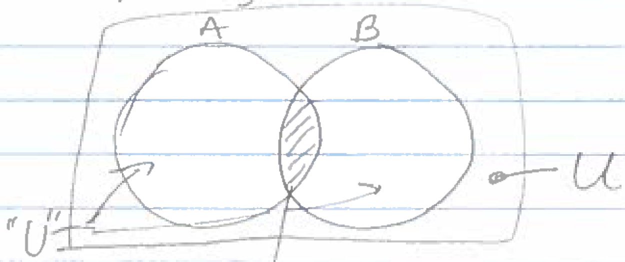
Set $A = \{1, 2, 3, 4, 9, 13\}$

$B = \{4, 3, 5, 6, 9, 1\}$

U

$A \cup B = \{1, 2, 3, 4, 9, 13, 5, 6\}$
"or"

$A \cap B = \{1, 3, 4, 9\}$
"and" intersection



$U = \{0, 1, 2, 3, 4, 5, 6, 9, 10, 11, 13\}$ "∩" intersection

$\bar{A} = \{5, 6, 10, 11, 0\}$
complement
everything

subset

complement

$\overline{A \cup B} = \{0, 10, 11\}$

[Subset of real #s]

Natural #s (counting #s) 1, 2, 3, 4, ...

Whole #s 0, 1, 2, 3, 4 ... Start from 0

Integers #s ... −3, −2, −1, 0, 1, 2, 3 ...

Rational #s $= \{\frac{a}{b} \mid a, b \text{ are integers}, b \neq 0\}$

Irrational #s $= \pi, \sqrt{2}, e$ nonterminating, non repeating decimals

Real #s

√ radical

decimal number 39.4872136

round to 2 dec. places 39.49
四捨五入

Truncate to 2 dec. places 39.48
切り捨て

$2 - Y = 6$ 2 less Y
subtract

[Order of Ops]

1. Parens
2. Exps, √
3. multiple, divide from L to R order
4. +, − (add and subtraction) from L to R order

$3 \cdot \frac{5}{6} - \frac{1}{2}$

$\frac{5}{2} - \frac{1}{2} = \frac{4}{2} = 2$

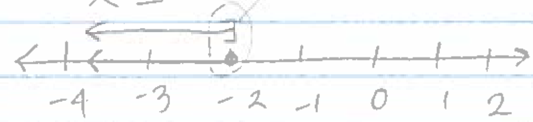
〈Section R 2〉 Algebra Essentials

[Graphing]

$5 < 7$

$13 \geq 13$

$x \leq -2$ includes end point



$d(P, Q) =$

$|-2-5| = |-7| = 7$ ← distance

distance

[Evaluating] $x = -2$, $y = 3$

$\frac{2x}{x-y} = \frac{2(-2)}{-2-3} = \frac{-4}{-5} = \frac{4}{5}$

[Domain of a Variable]

$$\frac{x}{x^2-25} = \frac{x}{(x+5)(x-5)}$$

$x \neq -5, 5$
± 5

$\{x \mid x \neq \pm 5\}$
↑ such as

negative exponents

$$4^{-2} = \frac{1}{4^{2}} = \frac{1}{16}$$

positive

★ Important
$(-1)^{-2} = 1$
$-1^{2} = -1$

* Rules for Exps (p22)

① $x^m x^n = x^{m+n}$

② $\frac{x^m}{x^n} = x^{m-n}$

③ $(x^m)^n = x^{mn}$

④ $(xy)^n = x^2y^2$

⑤ $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}, \ y \neq 0$

⑥ $x^{-n} = \frac{1}{x^n}, \ x \neq 0, n > 0$

⑦ $\frac{1}{x^{-1}} = x^n, \ x \neq 0, n > 0$

⑧ $x^0 = 1, \ x \neq 0$

$$\frac{4x^{-2}(yz)^{-1}}{2^3x^4y} = \frac{4x^{-2}y^{-1}z^{-1}}{8x^4y} = \frac{1}{2x^2yzx^4y}$$

$$= \frac{1}{2x^6y^2z}$$

Evaluating $x=2, y=-1$

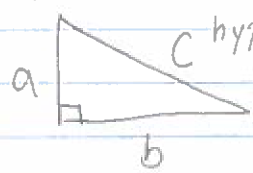
$$\sqrt{x^2} + \sqrt{y^2} = \sqrt{2^2} + \sqrt{(-1)^2} = \sqrt{4} + \sqrt{1} = 2+1 = 3$$

$(-1)^2 = 1$
$-1^2 = -1$

$3.4 \times 10^4 = 34000$
$1.694 \times 10^{-5} = 0.00001694$
$90,000,000 = 9 \times 10^{10}$
$906,000,000,000 = 9.06 \times 10^{11}$

Section R3: Geometry Essentials

pythagorean Thm.



hypotenuse – longest line

$$a^2 + b^2 = c^2$$

$a = 6$, $b = 8$, $c = 10$
36 64

distance, megoument positive

6, 4, 3 → this is a not size of right triangle.

$4^2 + 3^2 = 6^2$
$16 + 9 = 36$
$25 \neq 36$ no

☆

Section R4: Polynomials

多項式

polys $x^2 - 4x + 3$ → degree of each term

Not polys $3x^{-2}$
$1 + \frac{3}{r^2}$
$yt + x^{\frac{1}{2}}$

Ex] $9(y^2 - 3y + 4) - 6(1 - y^2)$
$9y^2 - 27y + 36 - 6 + 6y^2$
$15y^2 - 27y + 30$

$(5y+3)(2y-4)$
$= 10y^2 - 20y + 6y - 12$
$= 10y^2 - 14y - 12$

Ex. $(3x-2)^2$ 全部かける
$= 9x^2 - 12x + 4$

[Poly division]

$(5x^4 - x^2 + x - 2) \div (x^2 + 2)$

$5x^2 - 11 + \dfrac{x+20}{x^2+2}$

$x^2 + 0x + 2 \,\overline{)\, 5x^4 + 0x^3 - x^2 + x - 2}$
$\underline{5x^4 \mp 0x^3 \mp 10x^2}$
$-11x^2 + x - 2$
$\underline{+11x^2 \mp 0x + 22}$
$x + 20$

☆

Section 5: Factoring Polynomials

Grouping. 4項目 → 同じ項を見つける!

Ex. $3x^2 - 3x + 2x - 2$
$= 3x(x-1) + 2(x-1)$
$= (x-1)(3x+2)$

Ex. $x^2 - 10x + 16$
$(x-8)(x-2)$

$16 = -8 \times -2$
$-10 = \bigcirc + \bigcirc$

Ex. $2z^2 + 5z + 3$
$(2z+3)(z+1)$
$3z$, $2z$, $5z$

$2z^2 + 5z + 3$
$2z^2 + 2z + 3z + 3$
$2z(z-1) + 3(z+1)$
$(z-1)(2z+3)$

$c = -2$

$x - c \qquad x+2$

Ex. $8x^3 - 27$

$(2x-3)(4x^2+6x+9)$

☆ **Section R6: Synthetic Division**

Ex.) $(-4x^3+2x^2-x+1) \div (x+2)$

-2	-4	2	-1	$+1$
		8	-20	$+42$
	-4	10	-21	43 ← remainder

Answer: $-4x^2+10x-21+\frac{43}{x+2}$

Section R7: Rational Expressions

Dividing

Ex.)

$$\frac{\frac{x-2}{4x}}{\frac{x^2-4x+4}{12x}} = \frac{x-2}{4x} \times \frac{12x}{x^2-4x+4}$$

cross cancel

$x^2-4x+4 = (x-2)(x-2)$

$$= \frac{3}{x-2}$$ → three over $x-2$

Subtracting:

ex) $\frac{2x-3}{x-1} - \frac{2x+1}{x+1}$

least common:

LCM: $(x-1)(x+1)$

$$= \frac{(2x-3)(x+1)}{(x-1)(x+1)} - \frac{(2x+1)(x-1)}{(x-1)(x+1)} = \frac{2x^2-x-3}{(x-1)(x+1)} \quad \frac{2x^2-x-1}{(x-1)(x+1)}$$

$$= \frac{-2}{(x-1)(x+1)}$$

Simplefine complex rational expression

Ex.

$$\frac{1-\frac{x}{x+1}}{2-\frac{x-1}{x}} = \frac{\left(1-\frac{x}{x+1}\right)x(x+1)}{\left(2-\frac{x-1}{x}\right)x(x+1)}$$

$$= \frac{x(x+1)-x^2}{2x(x+1)-(x-1)(x+1)}$$

$$= \frac{x^2+x-x^2}{2x^2+2x-x^2+1}$$

$$= \frac{x}{x^2+2x+1} = \frac{x}{(x+1)^2}$$

答えはどちらでも良い

※ $x^3+y^3=(x+y)(x^2-xy+y^2)$

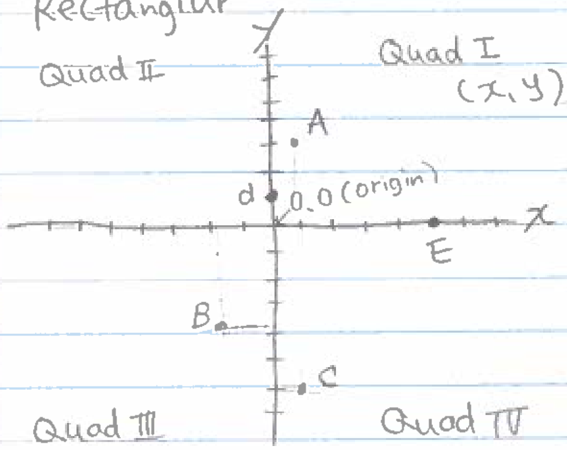
2013/9/16

Chapter 2

(2.1)

Distance and Midpoint Formula

Rectanglar

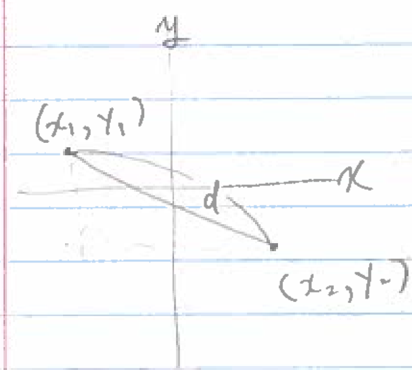


Plot these points

A = (1, 3)
B = (-2, -4)
C = (1, -6)
D = (0, 1)
E = (5, 0)

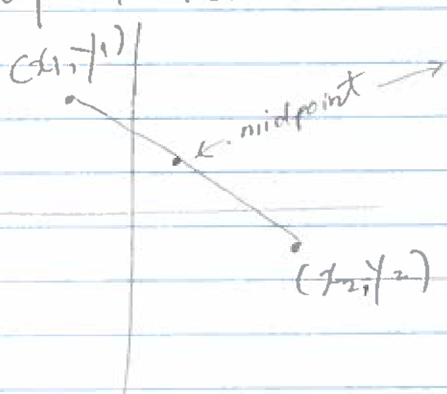
Distance Formula

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$



$a^2 + b^2 = c^2$

Midpoint Formula



$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

9/16/13

2-1

Ex. Find the distance between $(-2, 2)$ and $(1, 1)$

$d = \sqrt{(-2-1)^2 + (2-1)^2}$

$= \sqrt{9+1}$

$= \sqrt{10}$

Ex. Find the midpoint $(2, -3)$ $(4, 2)$

answer should be location

$\left(\frac{2+4}{2}, \frac{-3+2}{2}\right)$

$\left(3, -\frac{1}{2}\right)$

2-2

Graphs of Equations in two variables; Intercepts; Symmentry

Graphs of equations

Graphs - all points (x, y) which satisfy a given eqn.

Ex. Do these points lie on the graph of $y = x^3 - 2\sqrt{x}$? $(0,0)$ $(1,1)$ $(1,-1)$

$(0,0)$ $0 = (0)^3 - 2\sqrt{0}$ Yes

$(1,1)$ $1 = (1)^3 - 2\sqrt{1}$

$1 = -1$ no

$(1,-1)$ $-1 = (1)^3 - 2\sqrt{1}$

$-1 = 1 - 2$

$-1 = -1$ Yes

Intercepts - points at which graph touches or crosses x or Y - axis

To find x-int, let $y=0$ + solve for x
y-int, let $x=0$ ...... for y

Recall : Graphing Lines by Plotting points
* we can use this method to graph non-linear equations also.

x	y

自由曲線

Ex. Find the intercepts + graph each Eqn. by plotting pts. (Label intercepts)

a) $5x+2y=10$

☆ x inter → let $y=0$

$5x+2(0)=10$

$5x=10$

$x=2$

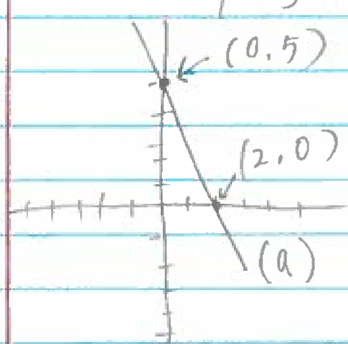
$(2,0)$

☆ y inter → let $x=0$

$5(0)+2y=10$

$2y=10$

$y=5$ $(0,5)$



b) $y=x^2-2$

☆ x inter let $y=0$

$0=x^2-2$

$x^2=2$

$x=\pm\sqrt{2}$

$=\pm 1.414......$

$(1.4,0)$ $(-1.4,0)$

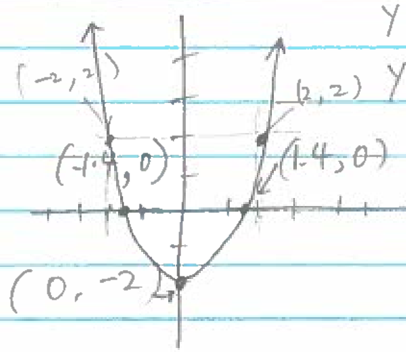
2つの答え.

☆ y inter → let $x=0$

$y=0-2$

$y=-2$

$(0,-2)$



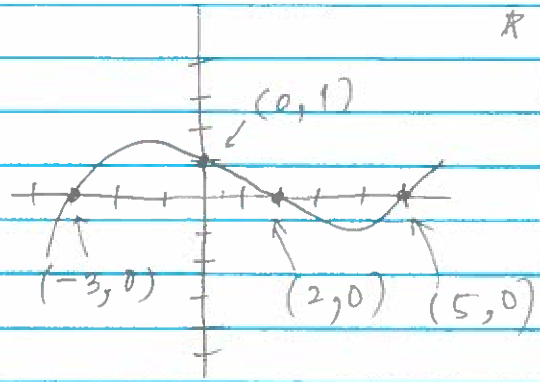
x	y
2	2
-2	2

2.2

parabola

- Reading Intercepts off of a Graph

Given:

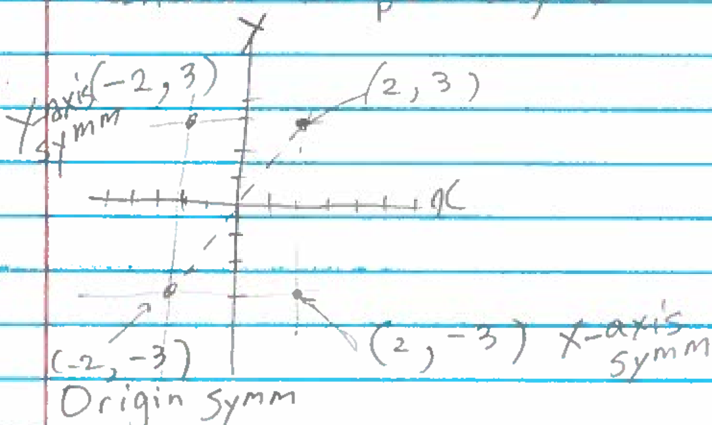


* Find intercepts

x-int	y-int
(-3,0)	(0,1)
(2,0)	
(5,0)	

- Symmentry - like a mirror image (reflection)

Consider the point (2,3)



- ~~Symmetry with Graphs~~

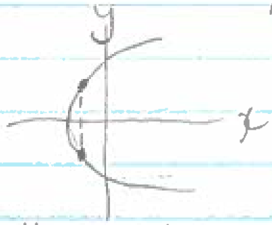
① x-axis symm: Graph looks like a reflection through x-axis
- for every point (x,y) on graph (x,-y) is also on graph

② Y-axis symm: graph looks like a reflection through y-axis
- for every point (x,y) on graph (-x,y) is also on graph

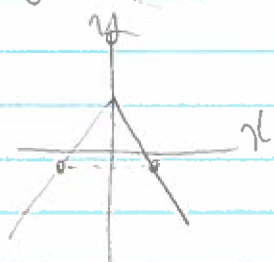
③ Origin Symm: graph looks like a reflection through Origin.
- for every point (x,y) on graph, (-x,-y) is also on graph

(2.2)

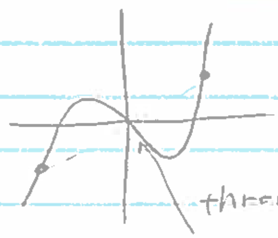
① x-axis symm



② y-axis symm



③ origin symm



p162 → Testing an Eq'n. for Symmentry

X-axis : replace y with −y + simplify
if you get an equiv Eq'n, then graph has X-axis symm.

Y-axis : replace x with −x + simplify
if you get an equiv. Eq'n. then graph has Y-axis symm

Origin : replace x with −x and y with −y + simplify.
if you get an equiv, then graph has origin symm

2.2

Ex) Test for Symm

a) $4x^2 + y^2 = 4$

- x-axis (yes)

$4x^2 + (-y)^2 = 4$

$4x^2 + y^2 = 4$

- y-axis (yes)

$4(-x)^2 + y^2 = 4$

$4x^2 + y^2 = 4$

- Origin (yes)

$4(-x)^2 + (-y)^2 = 4$

$4x^2 + y^2 = 4$

Important:

$$-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$$

b) $y = \frac{x^2 - 4}{2x}$

- x-axis (no)

$-y = \frac{x^2 - 4}{2x}$

- y-axis (no)

$y = \frac{(-x)^2 - 4}{2(-x)}$

$y = \frac{x^2 - 4}{-2x}$

- Origin (yes)

$-y = \frac{(-x)^2 - 4}{2(-x)}$

$-y = \frac{x^2 - 4}{-2x}$

$y = \frac{x^2 - 4}{2x}$

Ex) If $(-2, b)$ [x, y] is a pt. on the graph of $2x + 3y = 2$, what is b?

$2(-2) + 3(b) = 2$

$-4 + 3b = 2$

$3b = 6$

$b = 2$

1,

9/23   92   . notation

P167 $\{x \mid x \neq 3\}$ (2.3)

$a+bi \rightarrow$ still be the number

Quiz 60min + 90min
How

* Line

(slope) $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$, $x_1 \neq x_2$

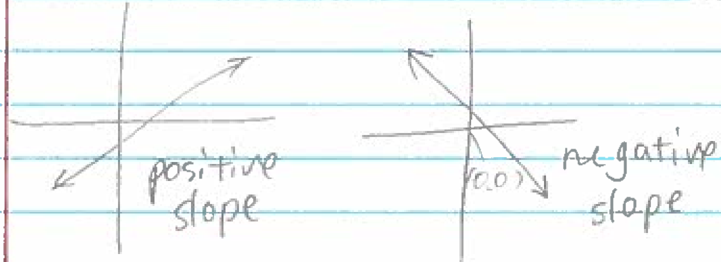
(x_1, y_1)
(x_2, y_2)

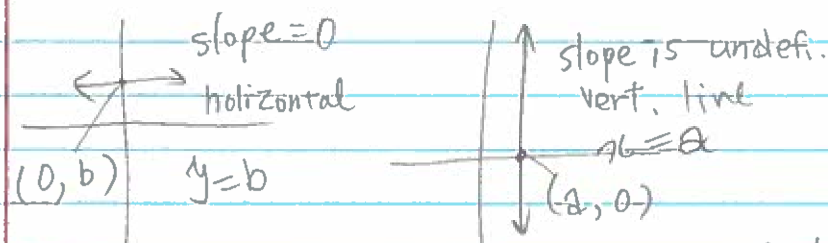
Ex) Find each slope

a) $(6,4)\ (-1,2)$ $m = \frac{4-2}{6+1} = \boxed{\frac{2}{7}}$

b) $(-3,0)\ (8,0)$ $m = \frac{0-0}{-3-8} = \frac{0}{-11} = \boxed{0}$

c) $(1,5)\ (1,-3)$ $\searrow$ $m = \frac{5+3}{1-1} = \frac{8}{0} = \boxed{\text{undefine}}$



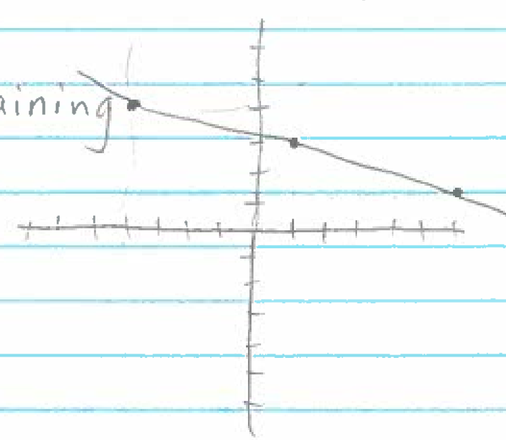


ex) Graph the line having

slope $= \frac{-2}{5}$ and containing the point $(1,3)$

$m = \frac{-2}{5} = \frac{2}{-5}$

(solution)



Ex) Equations of Lines

$y = b$ horizontal Line

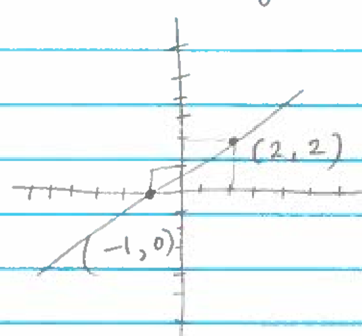
$x = a$ Vertical Line

** $y = mx + b$ Slope-Intercept Form

$y - y_1 = m(x - x_1)$ Point-Slope form

$Ax + By = C$ General Form

Ex) Find an equation of the line — find slope



Soln. $m = \frac{2-0}{2+1} = \frac{2}{3}$

$y = mx + b$

$0 = \frac{2}{3}(-1) + b$

$= -\frac{2}{3} + b$

$\frac{2}{3} = b$ ans. $y = \frac{2}{3}x + \frac{2}{3}$

$y - y_1 = m(x - x_1)$ — $y - y_2 = m(y - y_1)$

$y - 0 = \frac{2}{3}(x + 1)$

$y = \frac{2}{3}x + \frac{2}{3}$

Ex) Find an eqn. of the line with the given properties.

a) slope $= \frac{1}{2}$ (m) passing through $(3, 1)$

b) passing thru $(-3, 4)$ and $(2, 5)$

soln. ~~$y = mx + b$~~

$1 = \frac{1}{2}(3) + b$

$1 = \frac{3}{2} + b$

$-\frac{1}{2} = b$

$y = \frac{1}{2}x - \frac{1}{2}$

$(-3,4)$ and $(2,5)$

$y = mx + b$

First, Find slope

$$m = \frac{4-5}{-3-2} = \frac{-1}{-5} = \frac{1}{5}$$

$y = \frac{1}{5}x + b$

$4 = \frac{-3}{5} + b$

$\frac{20}{5} + \frac{3}{5} = b$

$\frac{23}{5} = b$

$$y = \frac{1}{5}x + \frac{23}{5}$$

$y - y_1 = m(x - x_1)$

$y - 4 = \frac{1}{5}(x - (-3))$

$y = \frac{1}{5}x + \frac{3}{5} + 4$

$$y = \frac{1}{5}x + \frac{23}{5}$$

Ex) Find the slope y-int + graph the line.

a) $3x + 2y = 6$ (General Form) b) $\frac{1}{2}y = x - 1$

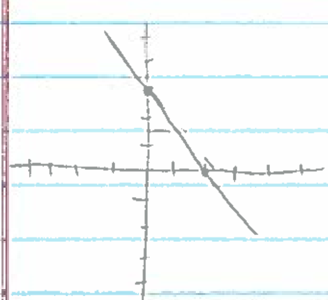
Soln. a) $2y = -3x + 6$

$y = -\frac{3}{2}x + 3$

Ans. $m = -\frac{3}{2}$, $b = 3$

Soln. b) $y = 2x - 2$

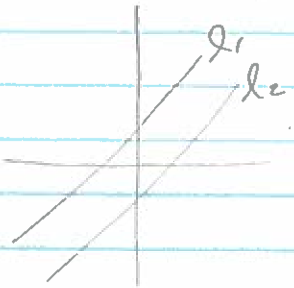
Ans. $m = 2$, $b = -2$



2.3

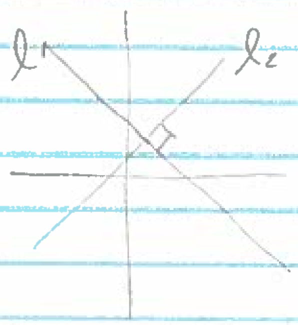
· <Parallel Lines>

- slopes are equal



$m_1 = m_2$

<Perpendicular Lines>



- slopes are opposite reciprocals
- their product is -1

flip over + and − with fraction

分母と分子を入れかえて プラス/マイナスを入れ替える.

⊥

m_1	m_2
$-\frac{2}{3}$	$\frac{3}{2}$
4	$-\frac{1}{4}$
$\frac{1}{5}$	-5
0.1	-10

Ex) Are these lines ∥, ⊥ or neither?

$$\begin{cases} y = \frac{1}{2}x - 3 \\ y = -2x + 4 \end{cases} = \perp$$

Ex) Find an eqn. of the line that passes thru $(1, -2)$ and is ⊥ (Perpendicular) to the line $y = 2x - 3$

Soln. $m_\perp = \frac{-1}{2}$ $\quad (1, -2)$

$y - y_1 = m(x - x_1)$

$y + 2 = -\frac{1}{2}(x - 1)$

$y = -\frac{1}{2}x + \frac{1}{2} - \frac{4}{2}$

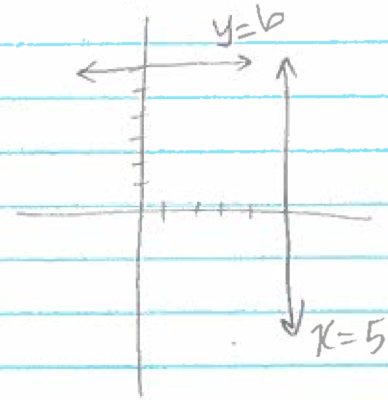
$y = -\frac{1}{2}x - \frac{3}{2}$

5, 137 (2.3) (2.4)

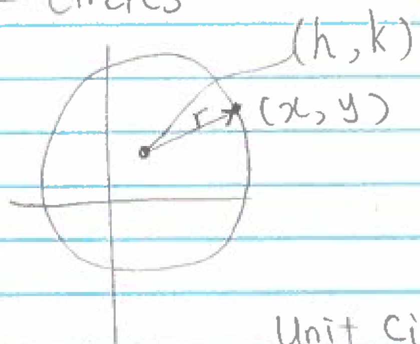
Ex) Find an eqn. of the line that passes thru $(-3, 6)$ and is $\perp$ to the line $x = 5$.
vertical line

Soln. Need a horizontal line

$y = 6$



(2.4) Circles



Standard Form

$(x-h)^2 + (y-k)^2 = r^2$

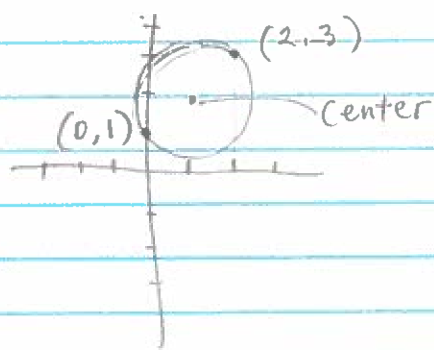
Center @ origin

$x^2 + y^2 = r^2$

Unit Circle,

$x^2 + y^2 = 1$

Ex) Find the center & radius of the circle ⊙ and write the equation in std. form.



$\left(\frac{2+0}{2}, \frac{1+3}{2}\right)$

Center $(1, 2)$

distance formula →

$r = \sqrt{(1-0)^2 + (2-1)^2}$

$r = \sqrt{2}$

(2.4) b,

Standard form

$$(x-h)^2+(y-k)^2=r^2$$

$$(x-1)^2+(y-2)^2=2$$

Step1, Center (h.k)

Step2, r

ex) Given center (2, -3) and r=4, find standard form.

General Form

$$x^2+y^2+ax+by+c=0$$

quadratic Turm — linear Turm — Constant

1. Find Standard Form
2. General Form
3. graph the ⊙ circle

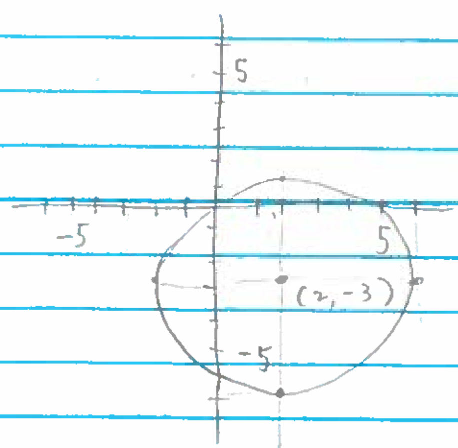
foil method ⟷ factoring method

Soln

Standard Form → $(x-2)^2+(y+3)^2=16$

$$x^2-4x+4+y^2+6y+9=16$$

General Form → $x^2+y^2-4x+6y-3=0$



7,

(3.1)

Ex.) Find the center + radius + graph the ⊙.

$x^2+y^2-6x+2y+9=0$

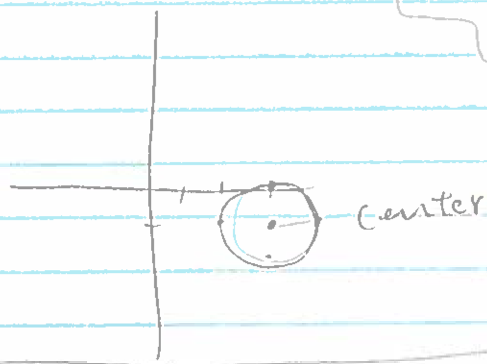
Solve) $(x^2-6x+9)+(y^2+2y+1)=-9+9+1$

$(x-3)^2+(y+1)^2=1$

↗ standard Form

Center $=(3,-1)$
$r=1$

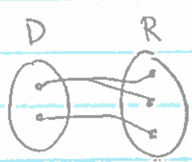
$9+y^2+2y+1=1$
$y^2+2y=-9$
$y^2+2y+9=0$



3.1 Functions

<u>Def.</u> function - a relation or correspondence or mapping from the set of input values (domain) to the set of out put values (range).

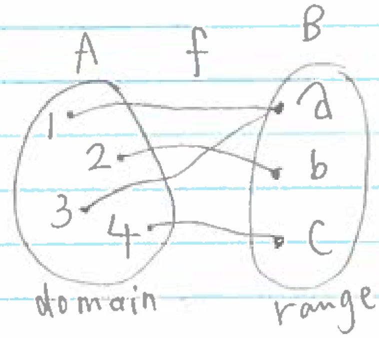
* Every element in the domain must be mapped to an element in the range.



not a function

Domain が枝分かれしていると not function

※ Note: One element in domain cannot be mapped to more than one element in range.



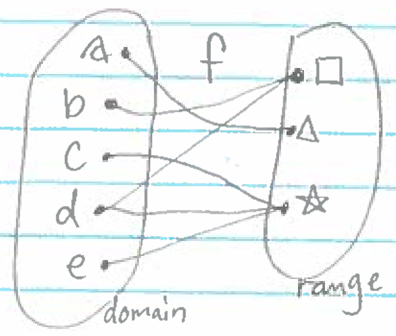
f is a function

distribute 8, 9

(3.1)

Ex) Are these relations functions?

a)



no

b) $\{(1,-2);(2,6);(3,0);(4,6)\}$

Yes

c) $y=x^3$

Yes

d) $x+y^2=1$

$y^2=1-x$

$y=\pm\sqrt{1-x}$

代入 $x=0$; $y=\pm\sqrt{1}$

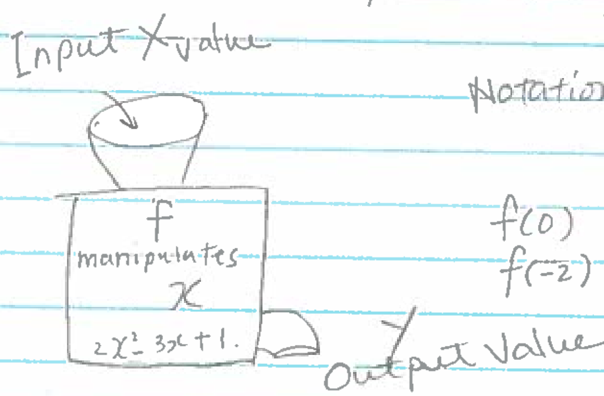
$y=\pm 1$

2つの答が出るので NO

y^2の場合は NO.

e) $y=\pm\sqrt{x^2-4}$

$y=\pm$の場合は NO



Notation $f(x)$ read of "f of x"

↑ input value

$f(0)$

$f(-2)$

$f(x+1)$

$f(t)$

Ex) let $f(x)=3x^2-x$. Evaluate the following.

a) $f(1)$

b) $f(-2)$

c) $f(a)$

d) $f(x+1)$

Soln. a) $f(1)=3(1)^2-1$

$=2$

b) $f(-2)=3(-2)^2+2$

$=14$

c) $f(a)=3(a)^2-a$

$=3a^2-a$

d) $f(x+1)=3(x+1)^2-(x+1)$

$=3(x^2+2x+1)-x-1$

$=3x^2+6x+3-x-1$

$=3x^2+5x+2$

9, (3.1)

分母: denominator
分子: numerator

Finding Domains

Domains – where function is defined
– those x values that make sense in function

① Polynomial functions – all $\mathbb{R}$ (real numbers)

$2x+3$

x^2+x+1

$2x^{\frac{1}{2}}, x^{-3}, \frac{1}{x}$ not real numbers

② Rational Functions –
– all real #s except where domain 0

③ Square Root Functions – all x values such that
(even n^{th} root fcns) the radicand ≥ 0
whats under $\sqrt{}$ ≥ 0
radical

Ex) Find the domain

a) $f(x) = \dfrac{x}{x^2-2x-24}$

$(x+4)(x-6)$

D: $x \neq -4, 6$

$(-\infty,-4) \cup (-4,6) \cup (6,\infty)$

b) $g(x) = 3x^4-2x^2+1$

D: all $\mathbb{R}$ $(-\infty, \infty)$

c) $h(x) = \sqrt{3x-1}$

$3x-1 \geq 0$

$3x \geq 1$

$x \geq \frac{1}{3}$ $[\frac{1}{3}, \infty)$

(3.1)

Operations on functions

(注)

足し算 – Sum $(f+g)(x) = f(x)+g(x)$

引き算 – Difference $(f-g)(x) = f(x)-g(x)$

かけ算 – Product $(fg)(x) = f(x)g(x)$

割り算 – Quotient $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, $g(x) \neq 0$

* Note, The domains of these functions are those x values in the domains of both $f+g$.

— 9/30/13 ———— (3.1) ———— 1,

Ex) Given $f(x) = \sqrt{x+1}$ and $g(x) = \frac{2}{x}$, find

a) $(f+g)(x)$ b) $(f-g)(x)$ c) $(fg)(x)$ d) $\left(\frac{f}{g}\right)(x)$

Soln: a) $(f+g)(x)$

Domain f: $\sqrt{x+1}$, $x+1 \geq 0$

$x \geq -1$ $[-1, \infty)$

Domain g: $\frac{2}{x}$, $x \neq 0$

$(f+g)(x) = \sqrt{x+1} + \frac{2}{x}$

Domain $\rightarrow \{x \mid x \in [-1, \infty)$ and $x \neq 0\}$

$[-1, \infty) = x \geq -1$

b) $(f-g)(x) = \sqrt{x+1} - \frac{2}{x}$

Important

$D: \{x \mid x \geq -1$ and $x \neq 0\}$

2,

逆数 → re...
レシプロカル

どちらもOK

c) $(fg)(x) = \frac{2}{x}\sqrt{x+1} = \frac{2\sqrt{x+1}}{x}$

$D: \{x \mid x \geq -1 \text{ and } x \neq 0\}$

d) $\left(\frac{f}{g}\right)(x) = \sqrt{x+1} \div \frac{2}{x}$

$= \sqrt{x+1} \times \frac{x}{2}$

$= \frac{x\sqrt{x+1}}{2}$

$D = \{x \mid x \geq -1 \text{ and } x \neq 0\}$

Ex) If $f(x) = 2x^3 + Ax^2 + 4x - 5$ and $f(2) = 5$, Find A.

$2(2)^3 + A(2)^2 + 4(2) - 5 = 5$

$16 + 4A + 8 - 5 = 5$

$4A = -14$

$A = -\frac{14}{4} \quad \frac{7}{2}$

$A = -\frac{7}{2}$

P214

The Graph of a Function

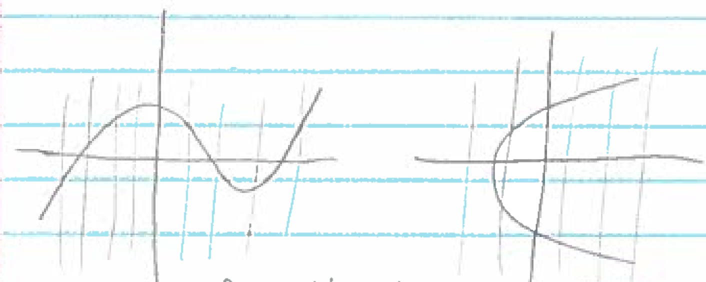
(3.2)

Use the vertical line test to determine if a graph is a function or not.

Vertical Line Test

there exists

Given a graph, if ∃ a vertical line that touches or crosses graph more than once, then graph is not a function.



<function> <not-function>

when write a vertical line, then the line cross/touch hit more than once, it is not function

Ex). Is this graph a fcn? * fcn = function

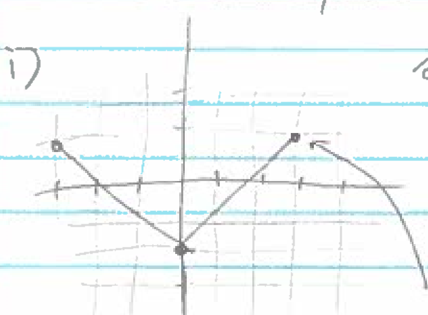
If so, what are its

a) domain + range

b) x + y — intersept

c) Is it symm with x, y, or origin?

i)

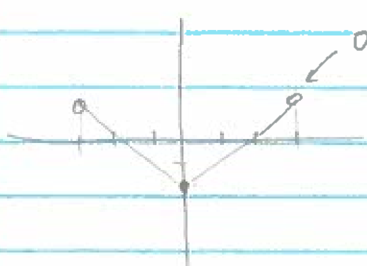


a) D: [−3, 3] ← { lowest x value, highest x value }

R: [−2, 1]

black dot の場合は < , > = [,],

open circle の場合は ≤, ≥, = (,),



(yes) this is function

4,

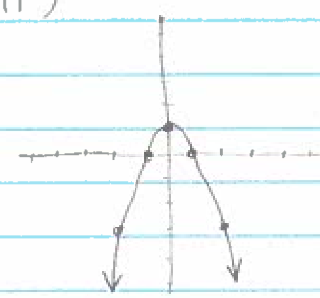
x value → Domain
y value → Range

i)

b) x-intersept y-int
(-2, 0) (0, -2)
(2, 0)

c) y-axis symmetry

ii)



– this is a function
a) D: $(-\infty, \infty)$
R: $(-\infty, 1]$
b) x-int , y-int
(1, 0) (0, 1)
(-1, 0)
c) y- axis symmetry

Ex) P 219
#26

$$f(x) = \frac{x^2+2}{x+4}$$

a) ~~Is the point~~ $\left(1, \frac{3}{5}\right)$ on the graph f ?
x : y

$$f(1) = \frac{(1)^2+2}{(1)+4} = \frac{3}{5}$$ yes

xに代入して、yの値が合うか確認

b) If $x=0$, what is $f(x)$?

$$f(0) = \frac{2}{4} = \frac{1}{2}$$

what is on the graph of f ?

$\left(0, \frac{1}{2}\right)$

Input output

d) What is the domain of f?

D: $x \neq -4$

$\{x \mid x \neq -4\}$

e) → x-int, let $y=0$

$0 = \frac{x^2+2}{x+4}$

Important

$\frac{3}{0}$ = undefine

$\frac{0}{3} = 0$

$0 = x^2 + 2$

$x^2 = -2$ → Imaginaly number (=ない).

$x = \pm\sqrt{-2}$ none — x-intersept is none.

f) → Y-int, let $x=0$

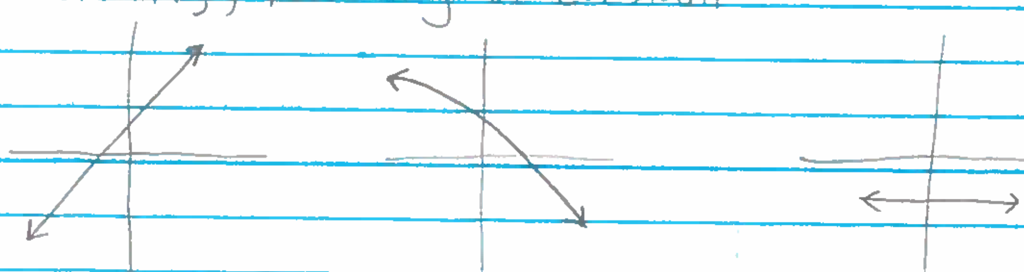
$y = \frac{0^2+2}{0+4} = \frac{1}{2}$ $\left(0, \frac{1}{2}\right)$

P218 #10 reading a graph

3.3

Properties of Functions

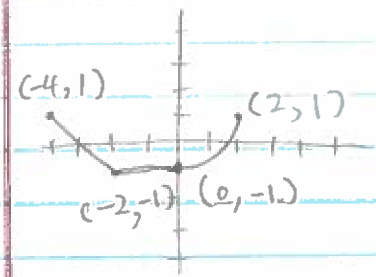
Increasing, Decreasing or Constant



increasing (右肩上がり) decreasing (右肩下がり) Constant (水平)

6,

Ex) On what intervals is the function ↑, ↓, ↔ ?



Soln. answer must in interval.

↓ (−4, −2)

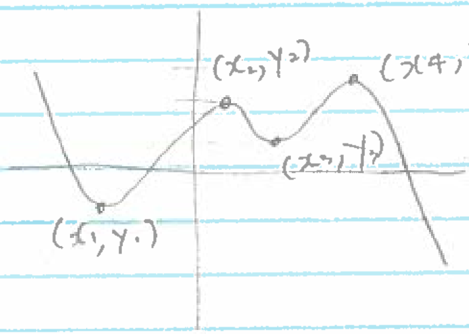
↔ (−2, 0)

↑ (0, 2)

→ [,] でも良い

→ Local Minimum of Local Maximum

– Peaks and Valleys



x-values

– #s at which function has a local max.

x_2 and x_4

– what are those local max values?

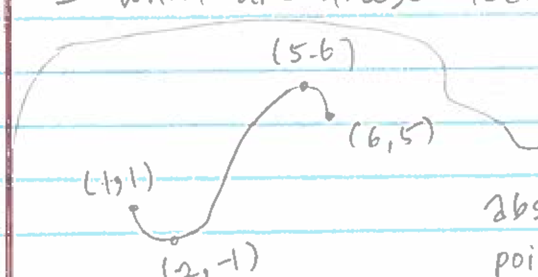
y_2 and y_4

– #s at which fcn has a minimum value

x_1 and x_3

– what are those local minimum values?

y_1 and y_3



absolute max

point at which fcn. has an abs. max.

$x = 5$

what's the absolute max value?

6

7,

(3.3)

Properties of Functions

Even of Odd Fcns - fcns only
- cannot be both
- can be neither

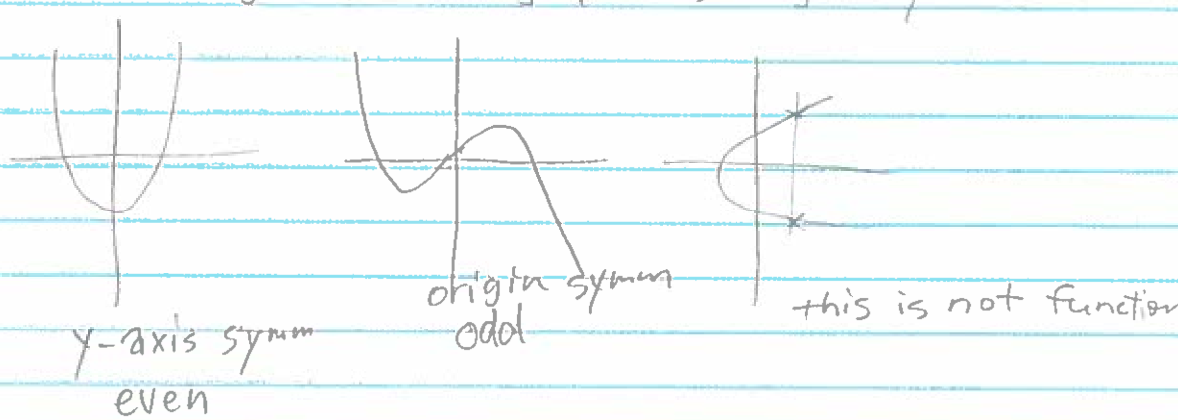
Even fcn - means fnc is symm with respective to the y-axis.

Odd fcn - 〃 to the origin.

Recall Test for symm

y-axis replace x with $-x$ of simplify
- if you got an equation equivarent to origin, then graph has y-axis symm.

Origin replace x with $-x$ and y with $-y$ a simplify
- if you get an equivarent equation to original, then graph has origin symm.



Ex) Determine algebraically whether the fcn is even, odd or neither.

a) $f(x) = 2x^4 - x^2$

Soln. y-axis symm test
(ネガティブ $-x$ を代入)

$y = 2(-x)^4 - (-x)^2$
$y = 2x^4 - x^2$

y-axis symm
even

8, 9

Secant?
secant line

b) $f(x) = x^5 - x + 1$

(−x を挿入) y-sym: $y = (-x)^5 - (-x) + 1$

$y = -x^5 + x + 1$ (no)

(−y, −x を入れ) org-sym: $-y = x^5 - x + 1$

$-y = (-x)^5 - (-x) + 1$

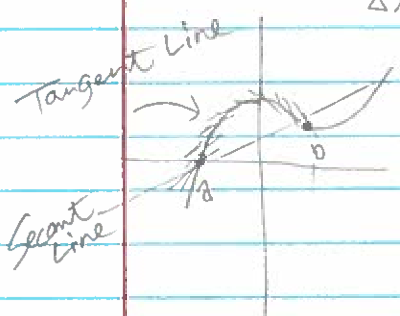
$-y = -x^5 + x + 1$

$y = x^5 - x - 1$ (no)

[neither]

– Average Rate of Change (of a function)

$$\frac{\Delta y}{\Delta x}$$



Slope of the Secant Line

– The Average rate of change of function from a to b equals the slope of the secant line containing the two points (a, f(a)) and (b, f(b)) on its graph.

☆ If the function $y = f(x)$ is defined at a and b, then the average rate of change of f between a and b is

(Same as slope)

$$\frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}, \quad a \neq b$$