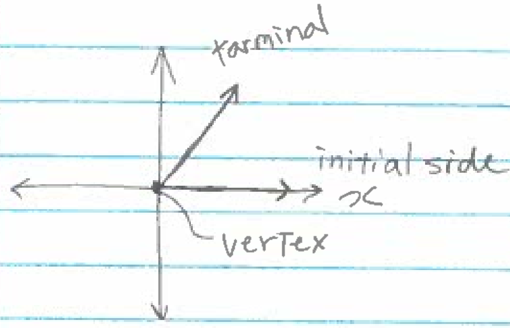
1/13/14

1,

<7.1> Angle and their Measure

Angle: Consists of an initial side and a terminal side.
The angle is identified by its rotation,
clockwise negative and counterclockwise is positive.



Standard position: initial side lies on the pos. x-axis
vertex is on the origin

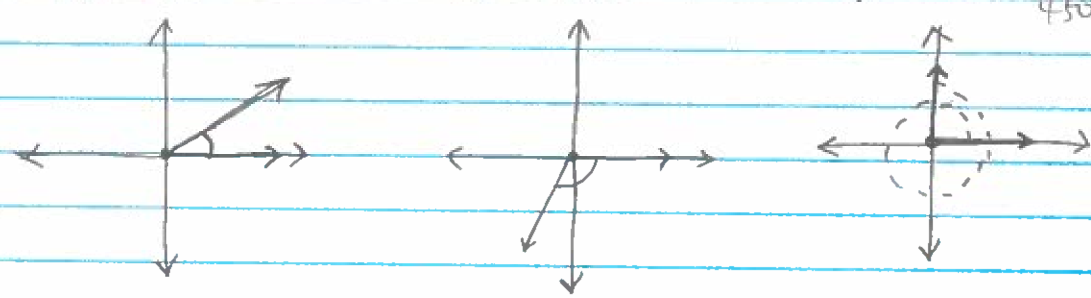
one revolution : 360°

$1° = 1/360$ of a revolution

ex) Draw each angle

① 30° ② −120° ③ 450°

$450-360 = 90$



— Converting to degrees, minutes, seconds

1 minutes $= 1' = \frac{1}{60}°$

1 second $= 1'' = 1/60'$

$1'' = 1/3600°$

Convert to angle in degrees.

④ $40°\ 10'\ 25''$

$40° + 10' + 25'' = 40° + 10(1/60)° + 25(1/3600)°$

$= 40 + 1/6° + 1/144° =$

$= 40 \frac{25}{144} = 40.17^\circ$

Convert the angle to D° M′ S″

$61.24^\circ = 61^\circ + .24(60)'$

$= 61^\circ + 14.4'$

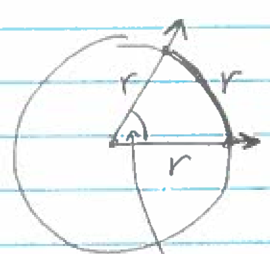
$= 61^\circ + 14' + .4(60)''$

$= 61^\circ + 14' + 24''$

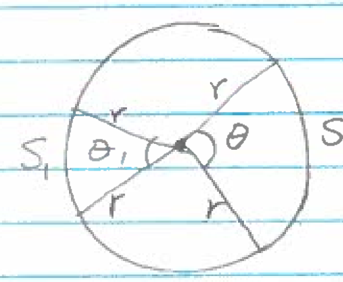
$= 60^\circ 14' 24''$

rays pass ~~thru~~ ~~the~~ circle

Radians: the measure of an angle subtended at the center of a circle by an arc that is equal in length to the rad



length of an arc of a circle.



$$\frac{\theta}{\theta_1} = \frac{S}{S_1}$$

let $\theta_1 = 1$ radian, then $S_1 = r$

$$\frac{\theta}{1} = \frac{S}{r}$$

$$S = \theta r$$

θ is an angle in radians, S is the arc length, r is the radius.

Ex) Find the missing value.

① $\theta = 1/3$ rad.
$S = 2$ ft

$2\text{ft} = 1/3 \text{ rad} (r)$
$r = 6$

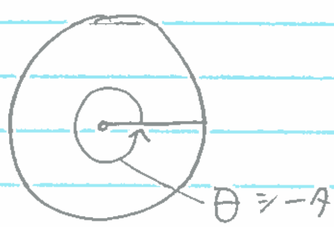
② $r = 6$m.
$s = 8$m

$8 = \theta 6$
$\theta = 4/3$ rad.

③ $r = 2$ in
$\theta = 30°$

can't do it because it is degree

— Convert degrees to radians + vise versa



Circumference $= 2\pi r = \theta r$

$2\pi = \theta$

$2\pi = 360°$ (1 revolution)

$\pi = 180°$ (½ revolution)

$$\boxed{\begin{array}{l} 1° = \pi/180 \text{ rad.} \\ 1 \text{ rad} = \dfrac{180}{\pi}° \end{array}}$$

— Convert degrees to radians

$$30° \times \frac{\pi}{180} = \frac{30\pi}{180} = \frac{\pi}{6} \text{ rad.}$$

$$-135° \times \frac{\pi}{180°} = \frac{-3\pi}{4} \text{ rad.}$$

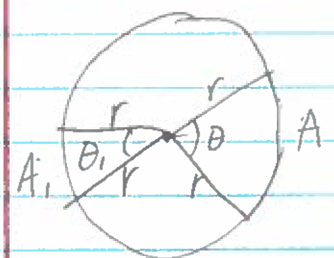
4,

- Convert radians to degrees.

$$\frac{5\pi}{6} \times \frac{180^\circ}{\pi} = 150^\circ$$

$$-4\pi \times \frac{180}{\pi} = -720^\circ$$

- Area of a sector



$$\frac{\theta}{\theta_1} = \frac{A}{A_1}$$

let $\theta_1 = 2\pi$ rad.

$A_1 = \pi r^2$

$$\frac{\theta}{2\pi} = \frac{A}{\pi r^2}$$

$A = \frac{1}{2} r^2 \theta$, θ is the angle in <u>Radians.</u>

(not degrees!)

linear Speed : an object moves around a circle of radius r at a constant speed. If S is the distance traveled in t seconds, then the linear speed is

$$V = \frac{S}{t}.$$

Angular Speed: ω (omega) $= \frac{\theta}{t}$

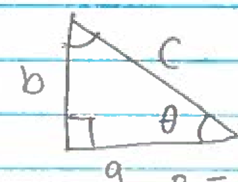
$$V = r\omega$$

1/15/14

<7.2> Right Triangle Trigonometry

Pythagorean Theorem : If a triangle is a right triangle, then $c^2 = a^2 + b^2$ where $a + b$ are length and C is the hypotenuse

— 6 ratios with right triangles



θ = acute angle

☆ $\sin\theta = \dfrac{opp}{hyp} = \dfrac{b}{c}$

☆ $\cos\theta = \dfrac{Adj}{hyp} = \dfrac{a}{c}$

☆ $\tan\theta = \dfrac{opp}{adj} = \dfrac{b}{a}$

☆ $\csc\theta = \dfrac{hyp}{opp} = \dfrac{c}{b}$
(cosecant)

☆ $\sec\theta = \dfrac{hyp}{adj} = \dfrac{c}{a}$

☆ $\cot\theta = \dfrac{adj}{opp} = \dfrac{a}{b}$

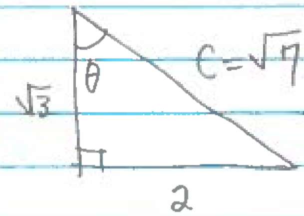
— Reciprocal Identities

$\csc\theta = \dfrac{1}{\sin\theta}$, $\sec\theta = \dfrac{1}{\cos\theta}$, $\cot\theta = \dfrac{1}{\tan\theta}$

— Quotient Identities

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Ex1) Find the values of the 6 trig Functions.



$c^2 = (2)^2 + (\sqrt{3})^2$

$c^2 = 4 + 3$

$c^2 = 7$

$c = \sqrt{7}$

$\sin\theta = \frac{2}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{2\sqrt{7}}{7}$

$\cos\theta = \frac{\sqrt{3}}{\sqrt{7}} = \frac{\sqrt{21}}{7}$

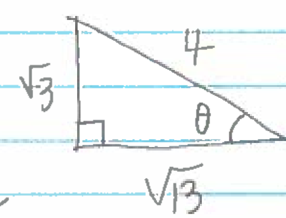
$\tan\theta = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\csc\theta = \frac{\sqrt{7}}{2}$

$\sec\theta = \frac{\sqrt{7}}{\sqrt{3}} = \frac{\sqrt{21}}{3}$

$\cot\theta = \frac{\sqrt{3}}{2}$

Ex2) $\sin\theta = \frac{\sqrt{3}}{4}$



$4^2 = (\sqrt{3})^2 + a^2$

$16 = 3 + a^2$

$13 = a^2$

$\sqrt{13} = a$

$$\cos\theta = \frac{\sqrt{13}}{4}$$

$$\tan\theta = \frac{\sqrt{3}}{\sqrt{13}} = \frac{\sqrt{39}}{13}$$

$$\csc\theta = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$$

$$\sec\theta = \frac{4}{\sqrt{13}} = \frac{4\sqrt{13}}{13}$$

$$\cot\theta = \frac{\sqrt{13}}{\sqrt{3}} = \frac{\sqrt{39}}{3}$$

Identities

$a^2+b^2=c^2$ divide by $c^2 \Rightarrow \frac{a^2}{c^2} + \frac{b^2}{c^2} = 1$

$$\Rightarrow \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = 1$$

$$\Rightarrow (\sin\theta)^2 + (\cos\theta)^2 = 1$$

$$\Rightarrow \sin^2\theta + \cos\theta^2 = 1$$

$\sin^2\theta + \cos\theta^2 = 1$, divided by $\cos^2\theta$

$$\Rightarrow \frac{\sin^2\theta}{\cos^2\theta} + 1 = \frac{1}{\cos^2\theta}$$

$$\Rightarrow \tan^2\theta + 1 = \sec^2\theta$$

$\sin^2\theta + \cos^2\theta = 1$, divided by $\sin^2\theta$

$$\Rightarrow 1 + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$\Rightarrow 1 + \cot^2\theta = \csc^2\theta$$

Find the exact value of each expression

$$\boxed{\cot 25^\circ} - \frac{\cos 25^\circ}{\sin 25^\circ} = \frac{\cos 25^\circ}{\sin 25^\circ} - \frac{\cos 25^\circ}{\sin 25^\circ} = 0$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Complementary Angle Theorem:

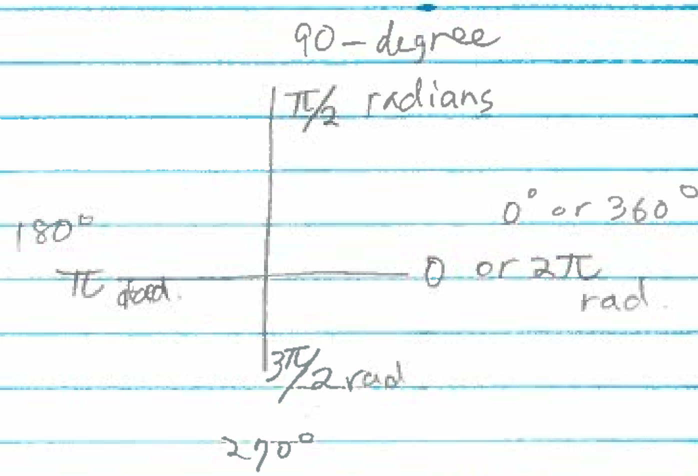
Cofunctions of complementary angles are equal

★ Angles (degrees)

$$\sin(\theta) = \cos(90^\circ - \theta)$$
$$\tan(\theta) = \cot(90^\circ - \theta)$$
$$\csc(\theta) = \sec(90^\circ - \theta)$$

★ Radians

$\sin(\theta) = \cos\left(\frac{\pi}{2} - \theta\right)$



Ex) Find the exact value

$\sin(38^\circ) - \cos(52^\circ)$

$\sin(38^\circ) - \sin(90^\circ - 52^\circ)$

$\sin(38^\circ) - \sin(38^\circ) = 0$

Ex) $\cot(25^\circ) \cdot \csc(65^\circ) \cdot \sin(25^\circ)$

$= \frac{\cos(25^\circ)}{\sin(25^\circ)} \cdot \frac{1}{\sin(65^\circ)} \cdot \sin(25^\circ)$

$= \frac{\cos(25^\circ)}{\sin(65^\circ)}$

$= \frac{\cos(25^\circ)}{\cos(25^\circ)} = 1$

Ex) Given $\tan\theta = 4$, Find each

① $\sec^2\theta = \tan^2\theta + 1$
$= (\tan\theta)^2 + 1$
$= 16 + 1$
$= 17$

② $\cot\theta = \frac{1}{\tan\theta} = \frac{1}{4}$

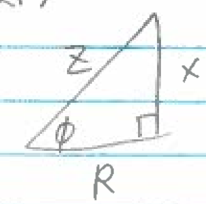
③ $\cot\left(\frac{\pi}{2} - \theta\right) = \tan(\theta) = 4$

<7.3>

(4) $\csc^2\theta = \cot^2 + 1$

$= \left(\frac{1}{4}\right)^2 + 1 = \frac{17}{16}$

1/22/14 21) Homework



ϕ is called the phase angle

$X = 800$ ohms

$R = 1200$ ohms

Find Z, $800^2 + 1200^2 = Z^2$

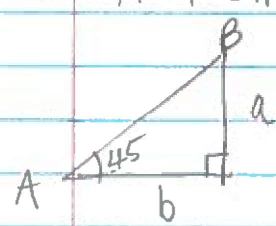
$Z = 1442.221$

$= 400\sqrt{13}$

Homework Quiz A+B due 1/30 !!!

<7.3> Computing the Values of Trigonometric Functions of Acute Angles

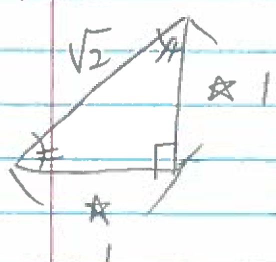
– Find all 6 trig function of $\frac{\pi}{4}$ (45°)



Since A = B, then a = b

The value of the trig. functions depend ons only on the angle and NOT the size of the triangle

you see



$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\tan\frac{\pi}{4} = \frac{1}{1} = 1$

Important

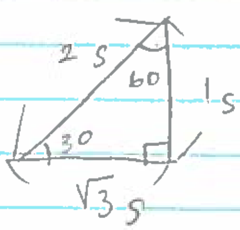
$\csc\frac{\pi}{4} = \sqrt{2}$

$\sec\frac{\pi}{4} = \sqrt{2}$

$\cot\frac{\pi}{4} = 1$

Important

- Find the exact values of the trig functions of $\frac{\pi}{6}$ (30°) and $\frac{\pi}{3}$ (60°)



$\frac{\pi}{6}$ (30°)

$\sin\frac{\pi}{6} = \frac{1}{2}$

$\csc\frac{\pi}{6} = \frac{2}{1} = 2$

$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\sec\frac{\pi}{6} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\tan\frac{\pi}{6} = \frac{3}{\sqrt{3}} = \frac{\cancel{3}\sqrt{3}}{\cancel{3}} = \sqrt{3}$

$\frac{\pi}{3}$ (60°)

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\csc\frac{\pi}{3} = \frac{2\sqrt{3}}{3}$

$\cos\frac{\pi}{3} = \frac{1}{2}$

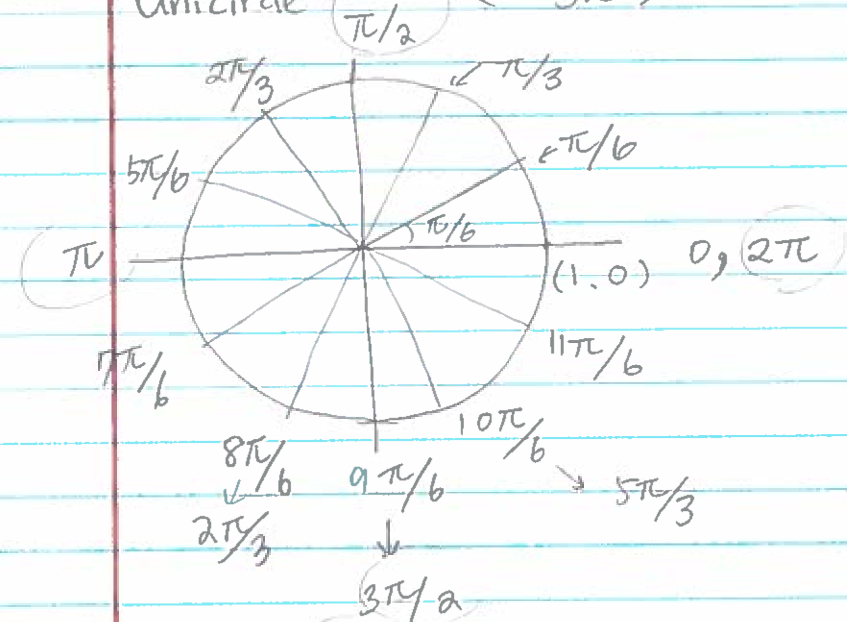
$\sec\frac{\pi}{3} = 2$

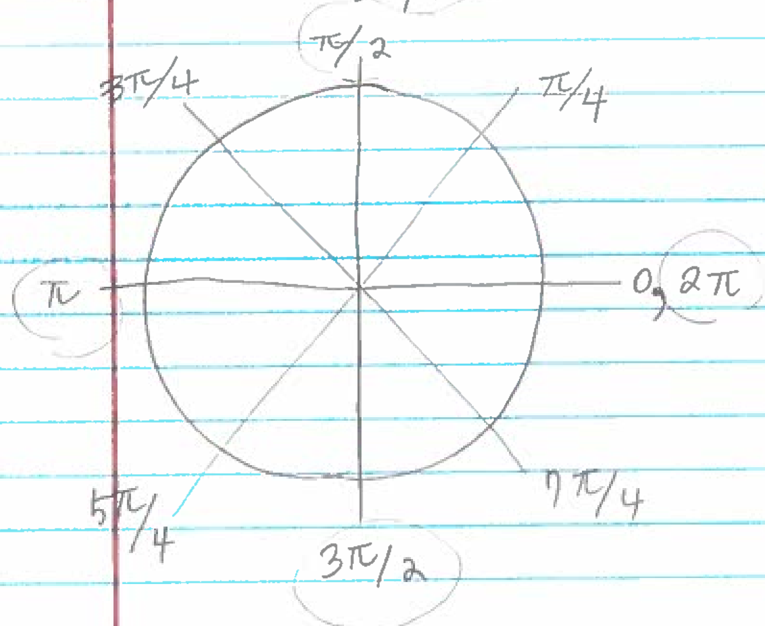
$\tan\frac{\pi}{3} = \sqrt{3}$

$\cot\frac{\pi}{4} = \frac{\sqrt{3}}{3}$

⟨7.3⟩

Unicircle ⟨angle⟩





Find the exact value of each.

① $f(\theta) = \sin\theta$, $\theta = 60°$

a) $f\left(\frac{\theta}{2}\right)$ $\sin 60°$

$f(30°) = \sin(30°)$
$= \frac{1}{2}$

b) $2f(\theta) = 2(\sin 60°) = \frac{\sqrt{3}}{2} = \sqrt{3}$

② $1 + \tan^2(30°) - \csc^2(45°)$

$1 + \left(\frac{\sqrt{3}}{3}\right)^2 - (\sqrt{2})^2 = 1 + \frac{1}{3} - 2 = -\frac{2}{3}$

③ $\sec\frac{\pi}{4} + 2\csc\frac{\pi}{3}$

$\sqrt{2} + 2\left(\frac{2\sqrt{3}}{3}\right) = \sqrt{2} + \frac{4\sqrt{3}}{3} = \frac{3\sqrt{2} + 4\sqrt{3}}{3}$

P537, #61

$t\theta = \sqrt{\frac{2a}{g \sin\theta \cos\theta}}$

b) $t(45°) = \sqrt{\frac{2(10)}{32 \sin(45°)\cos(45°)}}$

mistake?

$= \sqrt{\frac{20}{32\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)}} = \sqrt{\frac{40}{64 \times 4}} = \frac{40}{256}$

⟨7.3⟩

$$\sqrt{\frac{20}{32\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)}} = \left(\frac{20}{32\sqrt{\frac{2}{4}}}\right) = \sqrt{\frac{20}{16}} = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$$

a) $\theta = 30^\circ$

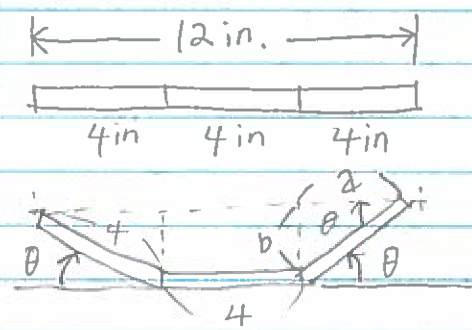
answer

$$t(30^\circ) = \sqrt{\frac{2a}{g\sin\theta\cos\theta}} = \sqrt{\frac{20}{8\sqrt{3}}} \approx 1.2\text{ sec}$$

$$= \sqrt{\frac{2(10)}{32(\quad)(\quad)}}$$

p532

Ex6) Area of Rectangle : $A = b \cdot 4 \longrightarrow 4(4\sin\theta)$



Area of the triangle:

$\sin\theta = \frac{b}{4} \rightarrow b = 4\sin(\theta)$

$\cos\theta = \frac{a}{4} \longrightarrow a = 4\cos(\theta)$

$$A = \frac{1}{2}(4\sin\theta)(4\cos\theta)$$

$$= 8\sin\theta\cos\theta$$

Area for two triangles : $16\sin\theta\cos\theta$

Total Area $= 16\sin\theta + 16\sin\theta\cos\theta$

$= \underbrace{16\sin\theta}_{GCF}(1 + \cos\theta)$

Ex6 つづき . b <7.3>

(b) find the area if $\theta = 30^\circ$

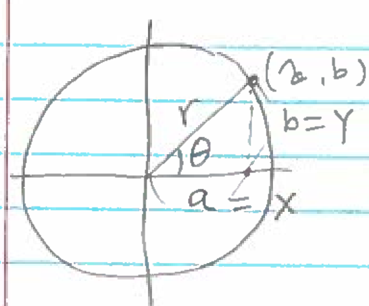
$$A = 16 \sin(30^\circ)(1+\cos 30^\circ)$$
$$= 16\left(\frac{1}{2}\right)\left(1+\frac{\sqrt{3}}{2}\right)$$
$$= 8\left(1+\frac{\sqrt{3}}{2}\right) = 8 + \frac{8\sqrt{3}}{2} = 8 + 4\sqrt{3} \approx 14.93 \text{ in.}$$

<7.4>

1/27 (Mon)

<Trigonometric Functions of Any Angle>

Defintion: Let θ any angle in standard position, and let (a, b) donate the coordinates of any point except $(0, 0)$, on the terminal side of θ.
If $r = \sqrt{a^2+b^2}$ denotes the distance from $(0,0)$ to (a,b), then the 6 trig. functions of θ are defined as:



$\sin\theta = b/r = Y/r$, $\csc = r/b = r/y$

$\cos\theta = a/r = x/r$, $\sec = r/a = r/x$

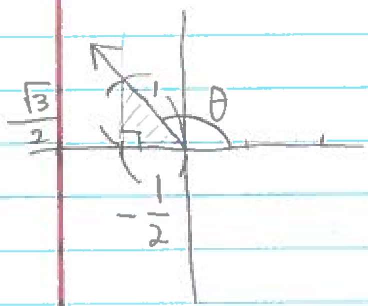
$\tan\theta = b/a = Y/x$, $\cos = a/b = x/Y$

provided no denominator is 0

$\neq 0$

⟨7.4⟩

Ex1) Find the values of all 6 trig. functions w/ $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ as a point on its terminal side.



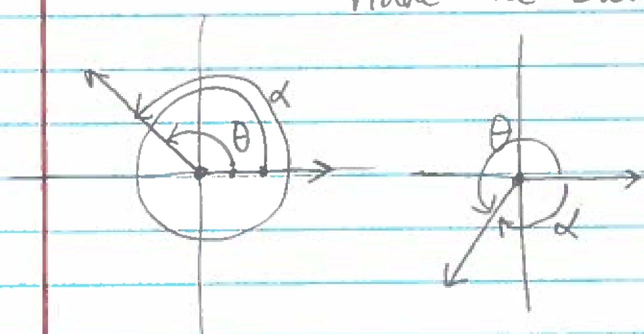
$$\left(-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2=\frac{1}{4}+\frac{3}{4}=1$$

$\sin\theta=\frac{\sqrt{3}}{2}$ $\qquad$ $\csc\theta=\frac{2}{\sqrt{3}}=\frac{2\sqrt{3}}{3}$

$\cos\theta=-\frac{1}{2}$ $\qquad$ $\sec\theta=-2$

$\tan\theta=\dfrac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}=-\sqrt{3}$ $\qquad$ $\cot\theta=\dfrac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}}=-\frac{1}{\sqrt{3}}=\frac{-\sqrt{3}}{3}$

Definition: Two Angles in standard position are called coterminal if they have the same terminal side.



$\alpha=\theta+K(2\pi)$ same

or $\alpha=\theta+K(360^\circ)$

$K\in\mathbb{Z}$

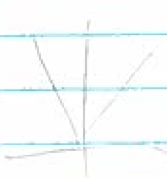
valuables

Real numbers $=\mathbb{R}$

Integers $=^{\oplus}\mathbb{Z}$ positive

$\in$

Ex2) Use coterminal angles to find the exact value of each



a) $\sin 405^\circ$

$405^\circ-360^\circ=45^\circ=\frac{\sqrt{2}}{2}$

b) $\cos\frac{33\pi}{4}$

$2\pi=\frac{8\pi}{4}$

$=\frac{\pi}{4}=\frac{\sqrt{2}}{2}$

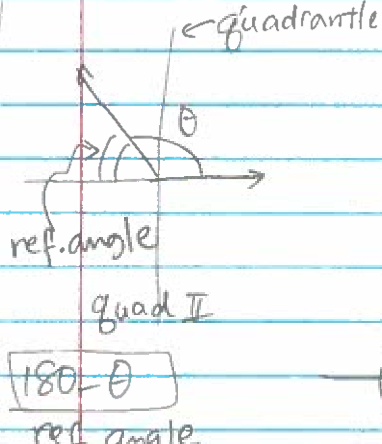
Ex3) Name quadrant in which the angle lies

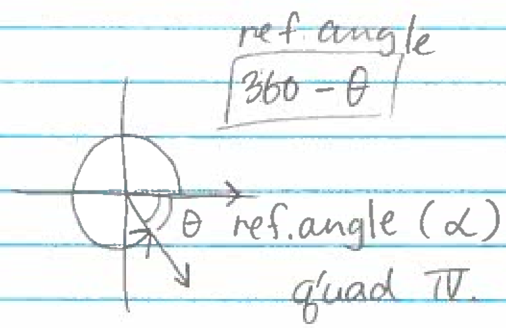
a) $\sin > 0,\ \cos < 0 \longrightarrow$ quadrant II

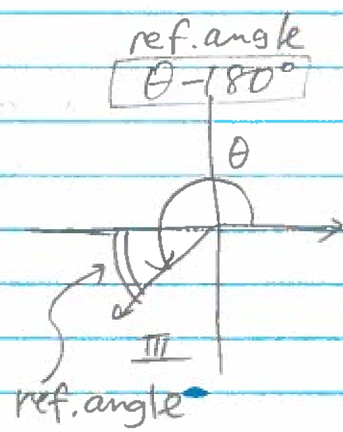
$(-,+)$ II	$(+,+)$ I
$(-,-)$ III	$(+,-)$ IV

	$\tan\theta$	$\cos\theta$	$\sin\theta$
I	+	+	+
II	−	−	+
III	+	−	−
IV	−	+	−

Definition: let θ denote an angle that lies in a quadrant. The acute angle formed by the terminal side of θ and the x-axis is called a reference Angle.







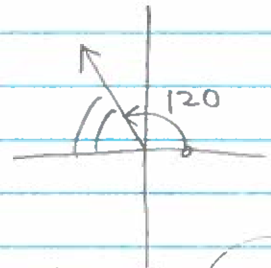
How do we find angle?

<7.4>

always positive

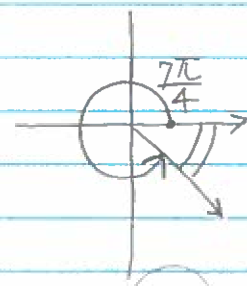
Ex4) Find the reference angles

a) 120°



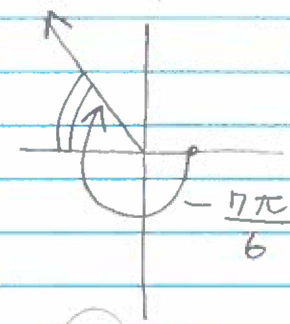
$180-120=60^\circ$

b) $\frac{7\pi}{4}$



$\frac{\pi}{4}=45^\circ$

c) $-\frac{7\pi}{6}$



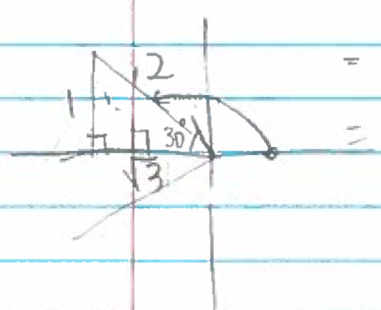
$\frac{\pi}{6}=30^\circ$

Theorem: If θ is an acute angle that lies in a quadrant and if α is its reference angle, then

$\sin\theta = \pm\sin\alpha$ ← True for all 6 trig. functions.

Ex5) use the reference angle to find each

a) $\sin 150^\circ$

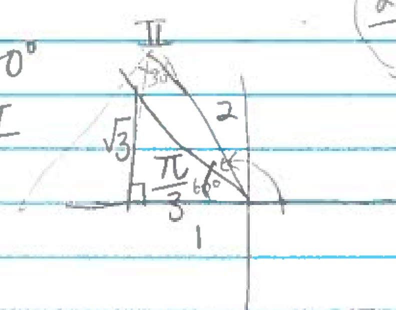


$=\pm\frac{1}{2}$

$=\frac{1}{2}$ because 180° is in Q II

b) $\tan\frac{14\pi}{3}$

$\frac{2\pi}{3}$



$\pm\sqrt{3}$

$=-\sqrt{3}$