Atwood’s Machine

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Objective:

Compares experimental to theory for constant force motion.

Theory:

Newton's second law indicates that the acceleration of an object depends on the net applied force and the object's mass as shown below.

$$F = ma \rightarrow a = \frac{F}{m}$$

The difference in weight between two hanging masses determines the net force acting on the system of both masses. This net force accelerates both of the hanging masses; the heavier mass is accelerated downward and the lighter mass is accelerated upward.

Assumptions: The mass of the pulley and string are ignored and the string has no stretch.

Figure 1 shows the setup of Atwood's Machine and free body diagrams of each mass. T is the tension in the string, and g is the acceleration due to gravity.

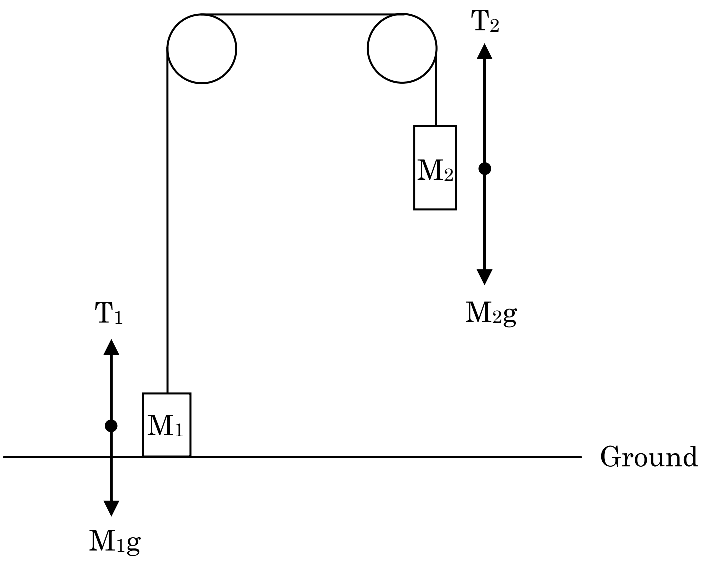


Figure 1: Atwood's Machine Settings

The equations of the each force are shown below.

$$T_1 - M_1 g = F_{net} = M_1 a \qquad \text{(Equation 1)}$$

$$T_2 - M_2 g = F_{net} = M_2(-a) \qquad \text{(Equation 2)}$$

Ideally, the tension forces for each mass are the same. Thus it can be set $T_1 = T_2$. Solving for the theoretical acceleration becomes the equation shown below.

$$a_{theory} = g \cdot \frac{M_2 - M_1}{M_2 + M_1} \quad \text{(Equation 3)}$$

For this experiment, measurement acceleration can be derived using the following equation.

$$a_{measured} = \frac{2d}{t^2} \quad \text{(Equation 4)}$$

d = distance M_2 traveled

t = duration M_2 traveled

Conclusion:

We compared experiment and theory for constant force motion.

For Constant Total Mass, I observed when the difference of those two masses becomes larger, the acceleration become larger in the case of constant mass.

For Constant Net Force, I observed when the sum of total mass becomes larger, the acceleration becomes smaller.

Overall, although the percentage error of the theory and experiment showed some concerns but those are thought of measurement error. Since the equation correlates to the experiment, I can say that we proved the theory is valid.

Procedure:

We conducted the experiment to derive the acceleration of both hanging masses using Atwood's machine. There are two measurements, constant total mass and constant net force.

Setup:

Mount the Static Board and attach two Pulleys on the board.

Connect two Mass hangers using a string and suspend over the two Pulleys.

Add masses at each end the smaller mass, M_1, and the larger mass, M_2.

The masses were varied depending on the purpose as shown below.

Constant Total Mass			Constant Net Force		
	M_1 (kg)	M_2 (kg)		M_1 (kg)	M_2 (kg)
Run #1	0.1295	0.1305	Run #6	0.055	0.060
Run #2	0.1290	0.1310	Run #7	0.060	0.065
Run #3	0.1285	0.1315	Run #8	0.065	0.070
Run #4	0.1280	0.1320	Run #9	0.070	0.075
Run #5	0.1275	0.1325	Run #10	0.075	0.080

Table 1: Settings of Mass

In order to set the constant total mass, we transferred the 0.5g of mass from the M_1 to the M_2 hanger.
In order to set the constant net force, difference of two masses stayed always 5g.

Pull the M_1 mass hanger to reach the ground and hold it until recording is set.

Measure the distance between the ground and the M_2.

Starting record:

Release the M_1 and let it fall and start measuring the time simultaneously. Stop the measurement time when the mass hits the ground to record duration of mass traveled.

Change the mass to record the different data and repeat.

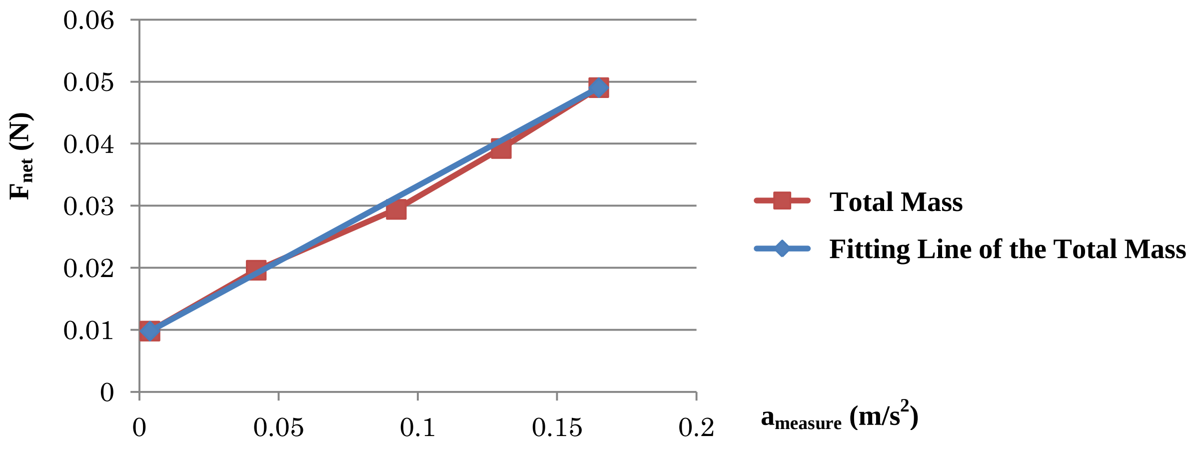
Data:

The distance from the ground to the M_2 was 44 inches = 1.1176m.

* Failed to obtain data Run #1 of Constant Total Mass

	Run	M_1 (kg)	M_2 (kg)	F_{net} (N)	M_1+M_2 (kg)	t_{ave} (s)	d (m)	$a_{measure}$ (m/s^2)	a_{theory} (m/s^2)	% difference
Constant Total Mass	#1	0.1295	0.1305	0.00981	0.26	N/A	1.1176	0.00385	N/A	N/A
	#2	0.1290	0.1310	0.01962	0.26	7.26	1.1176	0.042	0.0754	44.29
	#3	0.1285	0.1315	0.02943	0.26	4.92	1.1176	0.0923	0.113	18.32
	#4	0.1280	0.1320	0.03924	0.26	4.146	1.1176	0.130	0.1507	13.74
	#5	0.1275	0.1325	0.04905	0.26	3.68	1.1176	0.165	0.188	12.23
Constant Net Force	#1	0.055	0.060	0.04905	0.115	2.40	1.1176	0.388	0.426	8.92
	#2	0.060	0.065	0.04905	0.125	2.47	1.1176	0.366	0.392	6.63
	#3	0.065	0.070	0.04905	0.135	2.60	1.1176	0.331	0.363	8.82
	#4	0.070	0.075	0.04905	0.145	2.72	1.1176	0.302	0.338	10.7
	#5	0.075	0.080	0.04905	0.155	2.84	1.1176	0.277	0.316	12.3

Table 2: Collected Data



Graph 1: Graph of Constant Total Mass

Results:

The percentage difference was calculated using the following equation.

$$\text{Percentage error} = \frac{|M_{\text{theoritical}} - M_{\text{measured}}|}{M_{\text{theoritical}}} \times 100 \quad \text{(Equation 5)}$$

Graph 1 slope represents the total mass in kg as shown the following which related to Newton's second law. Newton's second law shows the relationship between the force, mass, and acceleration.

$$F = ma \rightarrow m = \frac{F}{a} \rightarrow \frac{(0.04905-0.00981)\text{kg}\cdot\text{m/s}^2}{(0.165-0.00385)\text{m/s}^2} = 0.243 \text{ kg} \quad \text{(Equation 6)}$$

Observation:

When the M_1 was released, M_2 didn't fall and stayed the same place for Run #1. It can be assumed that a friction force had occurred at the Pulleys.

The percentage error was larger in the case of Constant Total Mass compared to the Constant Net Force.

Analysis:

Constant Total Mass:

When the M_2 became heavier, M_1 became lighter. Then the difference of those two masses was larger. This situation about acceleration is expressed in the Equation 3. Using this equation, the values of acceleration for each run are shown in the table 2. I can say that when the difference of those two masses became larger, the acceleration became larger in the case of constant mass.

Constant Net Force:

When the sum of total mass larger, the acceleration smaller. This is because the difference of two masses stayed constant, and this difference of constant mass value smaller compared to the total mass on the system.

As shown in Graph 1, Newton's second law shows the relationship between the force, mass, and acceleration. Solving for mass, we used Equation 6 shown above. This means that if the mass is constant, the net force increases corresponding to the increase of acceleration, whereas when the net force is constant, the acceleration decrease corresponding to the increase of the net mass.

Percentage error for the mass in the system compared to the measurement data was 6.54%.

Error analysis:

There are some concerns which may have caused some numerical error.

1. Settings of the Atwood's machine

   Two Pulleys may not be set horizontally. We set those two Pulleys horizontally using a spirit level but it may not be accurate.

2. Friction Force on the Pulleys

   The friction force on the Pulleys might affect the net force. As mentioned in the Observation, we couldn't get data for Run #1 of Total Constant Mass. This is because the net force on the system was smaller compared to the friction force.

3. Recording Time

   When we recorded time from the mass release to reach the ground, we used stop a watch. This timing may not have been accurate. Also, the timing of the mass reaching the ground may not have been accurate.

4. Measurement of Length of Mass Traveled

   When we measured the length of the mass traveled, we used a meter stick and it may not perpendicular to the ground. This would cause numerical error since this value is used in Equation 4 which determines the acceleration from the measurement data.